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Dynamic Snap-Buckling of Shallow Arches under Inclined Loads

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Introduction

THE dynamic stability of shallow arches has been studied by many authors (see for example Ref. 1). Most of the works have been concerned with arches under uniformly distributed load. The stability of arches due to static concentrated load acting at the center of the arch is given by Dickie and Broughton.[2] The dynamic stability of arches subjected to concentrated eccentric load is studied in Refs. 1 and 3. In the present Note the dynamic snap-through of a clamped shallow circular arch by the action of a timewise step concentrated inclined load acting at an arbitrary point is studied. The material of the arch is assumed to be homogeneous, isotropic and linearly elastic.

Formulation and Solution

Referring to Fig. 1, the initial unstressed position of the arch is denoted by $y_0(x)$ and the displacements $w(x,t)$ are measured normal to x axis. Assuming that there is no initial thrust the equation of motion of the shallow arch can be written as

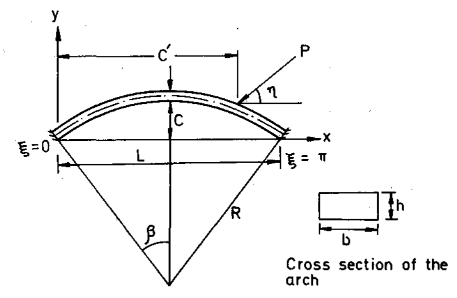

Fig. 1 Geometry of the arch.

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$$EI\frac{\partial^4 w}{\partial x^4} - H\frac{\partial^2(y_0-w)}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} + S(x-c')P\cos\eta\,\frac{\partial^2(y_0-w)}{\partial x^2} - P\sin\eta\,\delta(x-c') = 0 \qquad (1)$$

where

$$H = \left(1-\frac{c'}{L}\right)P\cos\eta + \frac{EA}{2L}\int_0^L\left[\left(\frac{dy_0}{dx}\right)^2 - \left(\frac{dy_0}{dx}-\frac{dw}{dx}\right)^2\right]dx$$

and

$$S(x-c') \quad = 1 \text{ if } x > c', \quad = 0 \text{ if } x < c'$$

Assuming a circular arch, and introducing nondimensional parameters

$$\phi = w/h, \quad \xi = \pi x/L, \quad \tau = t/R(E/\rho)^{1/2}, \quad \gamma = L^2/4hR,$$
$$\beta = L/2R, \quad e = \pi c/L$$

and

$$F = \pi P L^3/Ebh^4$$

Eq. (1) can be written as

$$\ddot{\phi} + \frac{\pi^4}{192\gamma^2}\phi^{iv} - \left(\phi'' + \frac{4\gamma}{\pi^2}\right)\left\{\frac{F\pi\beta}{2\gamma}\cos\eta\left(1-S(\xi-e)-\frac{e}{\pi}\right) - \frac{\pi^3}{32\gamma^2}\left[\int_0^\pi\left\{\phi'^2 - \frac{8\gamma}{\pi}\left(\frac{1}{2}-\frac{\xi}{\pi}\right)\phi'\right\}d\xi\right]\right\} - F\delta(\xi-e)\sin\eta = 0 \qquad (2)$$

where prime denotes $\partial/\partial\xi$ and dot denotes $\partial/\partial\tau$.

Solution is assumed in the form

$$\phi(\xi,\tau) = \sum_{n=1}^{\infty} a_n(\tau)\phi_n(\xi) \qquad (3)$$

where $\phi_n(\xi)$ are the eigenfunctions for normal modes of vibration of a clamped flat beam with corresponding eigenvalues m_n. Substituting Eq. (3) in Eq. (1), multiplying by ϕ_s, integrating from 0 to π and making use of the orthogonal property of ϕ_n, the following equations are obtained:

$$\ddot{a}_s + \frac{(m_s\pi)^4}{192\gamma^2}a_s + \frac{1}{A_s}\left(B_s + \frac{\pi^2}{4\gamma}\sum_{n=1}^{\infty}a_n D_{sn}\right) \times \left(\frac{\pi}{8\gamma}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_m a_n D_{mn} + \frac{B_s}{\pi}\sum_{n=1}^{\infty}a_n\right) + \frac{2F\beta\cos\eta}{\pi A_s} \times \left[\left(C_s - \frac{e}{\pi}B_s\right) + \frac{\pi^2}{4\gamma}\sum_{n=1}^{\infty}a_n\left(G_{sn} - \frac{e}{\pi}D_{sn}\right)\right] - \frac{F\sin\eta}{A_s}H_s = 0 \qquad (4)$$

Where A_s, B_s, C_s, D_{sn}, G_{sn} are definite integrals involving the eigenfunctions ϕ_n and $H_s = \phi_s|_{\xi=e}$. The amplitude $a_s(\tau)$ of the response can be calculated for different loads from the above sets of equations.

The critical snap buckling load is defined as that load for which a small increase in load produces a large increase in deflection. To arrive at this buckling load an average deflection ratio Δ defined as follows is introduced[4]:

$$\Delta(t) = \frac{1}{L}\int_0^\pi w(x,t)\,dx \Big/ \frac{1}{L}\int_0^\pi y_0(x)\,dx$$

and a value of Δ greater than unity indicates large deflection.

Numerical results and discussion

Equation (4) was integrated numerically using an IBM 7044 digital computer for a specified load (F), position of the load (e), inclination of the load (η), and maximum arch rise ($k = c/2h$) considering the first four terms in Eq. (3). The average deflection ratio (Δ) is calculated at different time τ. The load F and the time τ at which Δ becomes greater than 1 represents the buckling load and the time at which snap-through occurs.

The variation of the snap-through load with the maximum arch rise when $\eta = 30°$ is shown in Fig. 2. The snap-through load for different positions of the load for an arch rise $k = 3$ is plotted in Fig. 3. The effect of the inclination of the load also is shown. In the case when $\eta = 90°$ the snap load is symmetrical about the

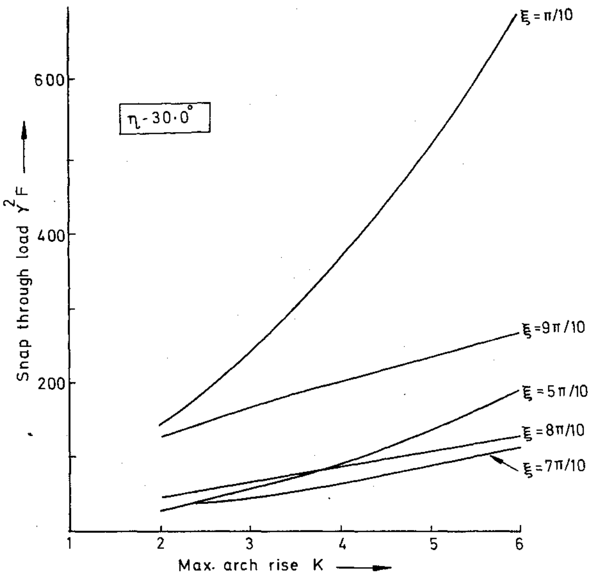

Fig. 2 $\gamma^2 F$ vs k for various load positions ($\eta = 30°$).

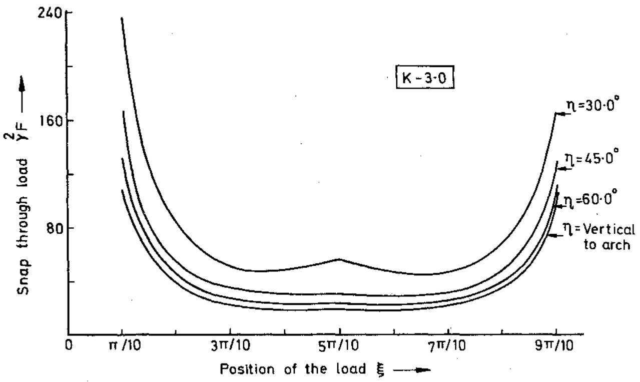

Fig. 3 $\gamma^2 F$ vs position of the load for various angle of inclination of the load η.

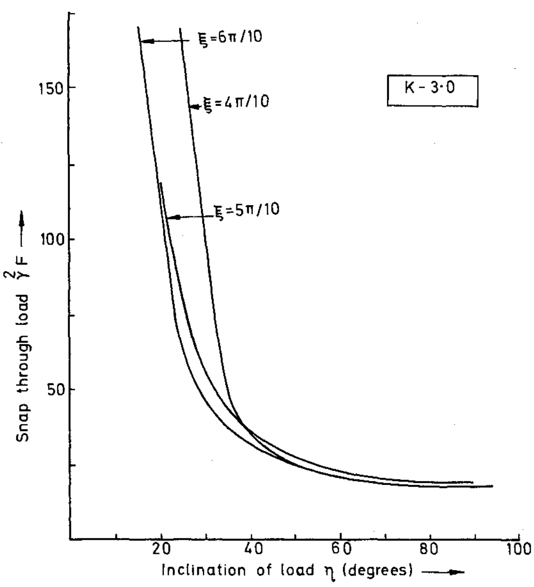

Fig. 4 $\gamma^2 F$ vs η for various position of the load.

center of the arch whereas this is not the case when the load acts at an inclination. The variation of the snap-through load with η for various positions of the load corresponding to the case when $k = 3$ is shown in Fig. 4. The snap-through load remains nearly a constant after a certain value of the inclination η.

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A Modified Gradient Technique for Solving Boundary and Initial Value Problems

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Nomenclature

a_k = polynomial constant coefficients
E_i = error of the ith equation
M = polynomial degree
N = number of strips for boundary-value problems
r = number of boundary conditions
x_j = generalized state variables

Introduction

When applying a numerical scheme to a system of nonlinear differential equations, they usually reduce to a system of nonlinear algebraic equations. There are a number of methods for solving nonlinear equations, such as, the shoot and hunt scheme, the perturbation method,[1,2] the method of quasilinearization,[3,4] and the method of accelerated successive replacements.[5]

The shoot and hunt method has the drawback of its sensitivity in some cases to the assumed initial values. The perturbation method depends mainly on the existence of small nonlinearity. For highly nonlinear equations, the method is questionable. The method of quasilinearization can not be applied successfully for systems producing ill conditioned matrices. On the other hand, the method of accelerated successive replacements alleviates the difficulties encountered in the other three methods. However, the number of iterations required to achieve a certain accuracy is considerably larger.[5]

The present analysis is intended to provide a simple and fast, yet accurate, method for solving boundary and initial value problems. Two applications are shown, the Blasius boundary-layer problem, which reduces to the solution of a highly nonlinear two point boundary value differential equation, and the problem of hypersonic flow over cones based on the hypersonic small disturbance theory.

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